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TITLE GEOMETRIC DATA FRUNCATION FOR FOURIER TRANSFORM PROGRAMS

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SOURCE LANGUAGE PAL III

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1. **Program Title:** Geometric Data Truncation for Fourier Transform Programs

2. **Abstract:** This program is intended for use as a truncating-weighting subroutine in conjunction with a Fourier Transform program. The application of a weighting function to the data record before the application of a Fourier transform program reduces the spread in frequencies which results from the transformation of a finite record. This procedure is often called apodization in that it removes the side lobes in the transformed record that results from a rectangular data window.

3. **Requirements:**
   3.1 **Storage:** The subroutine occupies 115(8) locations and is relocatable at the time of assembly.
   3.2 **Hardware:** PDP-8
   3.3 **Software:** The user must have a mainline program for gathering and storing in serial order the data before the application of this truncation algorithm. Following the weighting of the data the transform must be applied. The real series must be stored in serial order and without gaps. The length must be of the form $2^N$, just as required by the FFT. Additionally, the data must be in signed 11 bit, fixed-point format.

4. **Usage:** The program is intended to be assembled along with the mainline program. It is to be called after the data are gathered and before the actual FFT. The weighted data are replaced in their original locations with the original contents being lost. The subroutine is called with the number of data points in the AC and the address of the first point in CALL +1. Return is to CALL +2.

5. **Discussion:** When performing the discrete Fourier transform on a finite record in the time domain an inherent distortion results in the frequency domain. The FFT gives the power spectral density of an observed signal. Denoting Fourier transforms by $(\cdots)$ and convolutions by $(\ast)$,

$$A(v) = \mathcal{A}(t) \quad \text{and} \quad F(v) = \mathcal{F}(t)$$

where

$$A(v) = \int_{-\infty}^{\infty} A(t) e^{-i2\pi vt} \, dt$$
If $F(t)$ is the temporal representation of the observed signal, the power spectral density is given by

$$F(v) = \left| \int_{-\infty}^{\infty} F(t) e^{-i2\pi vt} dt \right| = \left| F(t) \right|$$

The data window here extends from $-\infty$ to $+\infty$ and weights each datum equally, i.e., $A(t) = 1$ for $-\infty < t < +\infty$.

The multiplication of two functions in the time domain is equivalent to convolving the transforms of the two functions in the frequency domain.

$$A(t) \cdot F(t) = A(t) \ast F(t) = A(v) \ast F(v)$$

$$= \int_{-\infty}^{\infty} A(v - v') F(v) dv'$$

For the case of an infinite data window

$$A(v) = A(t) = \int_{-\infty}^{\infty} 1 \cdot e^{-i2\pi vt} dt = \delta(v - v')$$

Hence the power spectral density

$$F(v) = \left| \int_{-\infty}^{\infty} F(t) e^{-2i\pi vt} dt \right| = \int_{-\infty}^{\infty} \delta(v - v') F(v) dv'$$

is the true power spectrum.

Infinite records are impossible and the observations of the temporal signal must be terminated. This limiting of $F(t)$ is truncation.

The most common form of truncation is the rectangular data window. Observations are made for a certain period of time and all information before and after this period is neglected. This gives rise to an $A(t)$ that is a rectangle of unit height. The Fourier transform, $A(v) = \frac{\sin \pi v}{\pi v}$, is a function with a finite spread and smaller side lobes, (see Fig 1A). The true spectrum is then convolved with a function which spreads each frequency into several frequencies.

Several other truncation functions have been used. These functions reduce the spread of the central peak
and/or remove the side lobes. Two of these are the triangular function with \( A(v) = \left( \frac{\sin \pi v}{\pi v} \right)^2 \) (see Fig. 1B).

and the Gaussian function with \( A(v) = e^{-\pi v^2} \), (see Fig. 1C). The application of these functions involves a degree of data handling that may be overly time consuming in order to obtain a clean power density spectrum.

This program involves a geometric truncation which requires no multiplications, or divisions, and yet gives a smooth truncated signal to be transformed. In the frequency domain the function to be convolved with the true spectral function has no side lobes and a minimal width.

6. **Description:** The data record is first divided into 16 blocks. The binary representation of each datum within a block is shifted to the right a number of times dependent upon the position of the block in the data record. The components of the blocks would be shifted 7, 6, 5, ..., 1, 0, 0, 1, 2, ..., 6, 7 places, resulting in multiplications of the components by \( 1/128, 1/64, 1/32, ...1/2, 1, 1, 1/2, 1/4, ... \), \( 1/64, 1/128 \) respectively. These factors are indicated by the horizontal bars in Fig. 2.

The truncation function when smoothed to its average value is the hyperbolic secant, \( \text{sech} (\pi t) \). The geometrical function has a number of discontinuities but at points of discontinuity the Fourier transform integral converges to the average

\[
\lim_{\varepsilon \to 0} \frac{1}{2} [A(t + \varepsilon) + A(t - \varepsilon)]
\]

of the right and left hand limits.

The Fourier transform of \( \text{sech} (\pi t) \) is \( \text{sech} (\pi v) \), (see Fig. 1D). So the function to be convolved with the true spectrum has no side lobes and minimal width.

7. **Execution Time:** The time required to weight a data record is \((100 \ \mu s) \times \text{(data record length)}\).

For example, a 512(10) point record length can be weighted in 0.0512 seconds.

8. **User Modification:** The user who intends to transform data records of a fixed length may wish to shorten the subroutine by calculating and storing some of the constants which would otherwise be recalculated and stored at each execution time.
9. **Listing:** A listing is below.

10. **Logic Flow Chart:** A logic structure flow chart follows the listing.

11. **References:**


Figure 1

1A: $\frac{\sin \pi x}{x}$

1B: $\left(\frac{\sin \pi x}{x}\right)^2$

1C: $e^{-\pi x^2}$

1D: $\text{sech} \, \pi x$
Truncation Function (Geometric & sech πx)
\textit{GENERALIZED AFOORIZATION FUNCTION}\hfill \\
\textit{E.A. PARNHARDT, 30 SEPTEMBER 1971}\hfill \\
\textit{FOR DATA OF LENGTH 2\textsuperscript{n}, THAT IS 1024, 512, 256, ETC}\hfill \\
\textit{RUNNING TIME, 100 MICROSEC TIMES DATA LENGTH}\hfill \\
\textit{DATA RESTORED TO ORIGINAL LOCATION.}\hfill \\
\textit{CALL WITH NUMBER OF DATA POINTS IN AC AND ADDRESS}\hfill \\
\textit{OF FIRST POINT IN CALL+1, RETURN IS TO CALL+2}\hfill \\
\textit{PROGRAM OCCUPIES 115(8) LOCATIONS}\hfill \\
\begin{align*}
4600 & 0000 \text{ AFOO,} \\
4601 & 3313 \text{ FCA PTS} \\
4602 & 1600 \text{ TAD I AFOO} \\
4603 & 3311 \text{ DCA ADP1} \\
4604 & 2200 \text{ ISZ AFOO} \\
4605 & 1313 \text{ TAD PTS} \\
4606 & 7012 \text{ FTS} \\
4607 & 7012 \text{ FTS} \\
4610 & 3302 \text{ FCA TEMPS} \\
4611 & 1302 \text{ TAD TEMPS} \\
4612 & 7001 \text{ CIA} \\
4613 & 3306 \text{ DCA CNTR3} \\
4614 & 1313 \text{ TAD PTS} \\
4615 & 1306 \text{ TAD CNTR3} \\
4616 & 1306 \text{ TAD CNTR3} \\
4617 & 7001 \text{ CIA} \\
4620 & 3307 \text{ DCA CNTR3} \\
4621 & 1313 \text{ TAD PTS} \\
4622 & 7010 \text{ FTS} \\
4623 & 1311 \text{ TAD ADD1} \\
4624 & 1302 \text{ TAD TEMPS} \\
4625 & 3312 \text{ DCA ADDP} \\
4626 & 1314 \text{ TAD 87} \\
4627 & 3303 \text{ DCA CNTR3} \\
4630 & 1306 \text{ LOOP1, TAD CNTR3} \\
4631 & 3310 \text{ DCA CNTR3} \\
4632 & 1711 \text{ LOOP1, TAD I ADD1} \\
4633 & 4265 \text{ JMS SFS} \\
4634 & 3711 \text{ DCA I ADD1} \\
4635 & 2211 \text{ ISZ ADD1} \\
4636 & 2307 \text{ ISZ CNTR4} \\
4637 & 2306 \text{ ISZ CNTR3} \\
4640 & 5232 \text{ JMP LOOP1} \\
4641 & 2303 \text{ ISZ CNTR3} \\
4642 & 5230 \text{ JMP LOOP1} \\
4643 & 3200 \text{ CLA CIL} \\
4644 & 7061 \text{ IAC} \\
4645 & 7041 \text{ CIA} \\
4646 & 3303 \text{ DCA CNTR3} \\
4647 & 1306 \text{ LOOP2, TAD CNTR3} \\
4650 & 3310 \text{ DCA CNTR3} \\
4651 & 1712 \text{ LOOP2, TAD I ADD2} \\
4652 & 4265 \text{ JMP SFS} \\
4653 & 3712 \text{ DCA I ADD2} \\
4654 & 2312 \text{ ISZ ADD2} \\
\end{align*}
CALL

Initialize:
number of data
1st half address
length of block
number of shifts

Get Datum
Shift Right

Shifting Completed?

No
Yes

Deposit Datum

Block Complete?

No
Yes

Increase Shift

Return

Next Block?

No
Yes

Initialize for second half

Get Datum
Shift Right

Shifting Completed?

No
Yes

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