# DECUS

## PROGRAM LIBRARY

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| DECUS NO. | 8-568 |
| TITLE | CFI - CONTINUED FRACTION INVERSION |
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| COMPANY | Capitol Institute of Technology<br>Kensington, Maryland |
| DATE | July 14, 1972 |
| SOURCE LANGUAGE | 8K FORTRAN |

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SUB 8K-# Jan. 10, 1972

(1) NAME: CFI Continued Fraction Inversion

Purpose: To convert a continued fraction into a rational function.

(2) CALLING SEQUENCE:

CALL CFI (N,H,B,A)

where:

Input Data:

N = one-half the no. of given H's (SEE (4) & (7) )
H(I),I = 1,2, ..., 2*N (SEE (4) & (7) )

Return Data:

B(I),I = 1,2, ..., N coefficients in numerator polyno.
A(I),I = 1,2, ...,(N+1) " " denominator "

(3) ERROR RETURN:

-none-

(4) SPECIAL CONSIDERATION:

See (7)

(5) SUBPROGRAM CALLED:

-none-

(6) STORAGE REQUIRED:

pages (octal)

(7) ALGORITHM & REFERENCES:

Given: $$G(s) = h_1 + \cfrac{1}{\cfrac{h_2}{s} + \cfrac{1}{h_3 + \cfrac{1}{\cfrac{h_4}{s} + \cfrac{1}{\cdots + h_{2n-1} + \cfrac{1}{\cfrac{h_{2n}}{s}}}}}}$$

Problem: Find $$G(s) = \frac{b_1 s^{n-1} + \cdots\cdots + b_n}{a_1 s^n + a_2 s^{n-1} + \cdots\cdots + a_{n+1}}$$

$a_1 = 1$

Algorithm:

Step 1: define $a_{ik} = \begin{array}{ll} 1 & \text{for } i = 1 \text{ , } k = 1,2, \ldots, n+1 \\ 0 & \text{for } i \text{ greater than } k = 1,2,\ldots,n+1 \end{array}$

$b_{ik} = 0$ if $i$ is greater than $k$

Step 2: For each $k = 2,3, \ldots, n+1$ compute for $i = 2,3,\ldots,k$

$$b_{ik} = h_{2(n-k)+4}\, a_{i-1,k-1} + b_{i,k-1}$$

$$a_{ik} = h_{2(n-k)+3}\, b_{ik} + a_{i,k-1}$$

Step 3: compute,

$$b_i = b_{i+1,n}$$

$$a_i = a_{i+1,n}$$

for $i = 1,2, \ldots, n$

References:


1. C.T. Chen, "A Formula and an Algorithm for Continued Fraction Inversion", Proc. IEEE(letters) vol. No. pp. 1780-1781, Oct. 1969.
2. C.F. Chen, L.S. Shieh, "Continued Fraction Inversion by Routh's Algorithm ", IEEE Trans Circuit Theory vol. CT-16, pp.197-202, May 1969.


(8) LISTING :

See attach.

(9) SAMPLE:

See attach.

```
C ****  " CONTINUED FRACTION INVERSION "
C ****    RESULTS IS A  RATIONAL FUNCTION
C ****    (B(1)*S**(N-1) + ... + B(N))/(S**N + A(1)*S**(N-1)+
C         ....+A(N))
C         JAN. 10'72
C         N IS ATMOST 10
      SUBROUTINE CFI(N,H,B,A)
      DIMENSION H(20),B(10),A(11),A1(11,11),B1(11,11)
      N1=N+1
      DO 10 J=1,N1
      A1(1,J)=1.
      J1=J+1
      DO 10 I=J1,N1
      A1(I,J)=0.
      B1(I,J)=0.
10    CONTINUE
      DO 20 K=2,N1
      DO 20 I=2,K
      NK=2*(N-K+2)
      B1(I,K)=H(NK)*A1(I-1,K-1) + B1(I,K-1)
      A1(I,K)=H(NK-1)*B1(I,K)+A1(I,K-1)
20    CONTINUE
      DO 30 I=1,N
      II=I+1
      B(I)=B1(II,N1)
      A(II)=A1(II,N1)
30    CONTINUE
      A(1)=1.
      RETURN
      END
```

```
C ****  SAMPLE # 1
C ***   RR = DUMMY ALPHANUMERIC DATA MUST BE A  " SPACE "
        DIMENSION H(14),B(7),A(8)
 3      READ(2,5) RR,N
5       FORMAT(A1,I10)
        IF (N) 100, 100, 10
10      NN=2*N
        DO 20 I=1,NN
20      READ(2,15) H(I)
15      FORMAT(E10.4)
C
        CALL CFI(N,H,B,A)
        WRITE(2,35)RR,
        DO 30 I=1,N
30      WRITE(2,15) B(I),
        FINI
35      FORMAT(//9X,A1)
        N1=N+1
        DO 40 I=1,N1
40      WRITE(2,15) A(I),
        FINI
        GO TO 3
100     STOP
        END
```

1
1.
2.
2
1.
2.
3.
4.
3
1.
2.
3.
4.
5.
6.
4
1.
2.
3.
4.
5.
6.
7.
8.
5
1.
2.
3.
4.
5.
6.
7.
8.
9.
10.
2
1.
2.
0.
4.
2
1.
0.
0.
4.
2
3.
1.1429
9.8
.3571
2
1.
0.
0.
0.
0

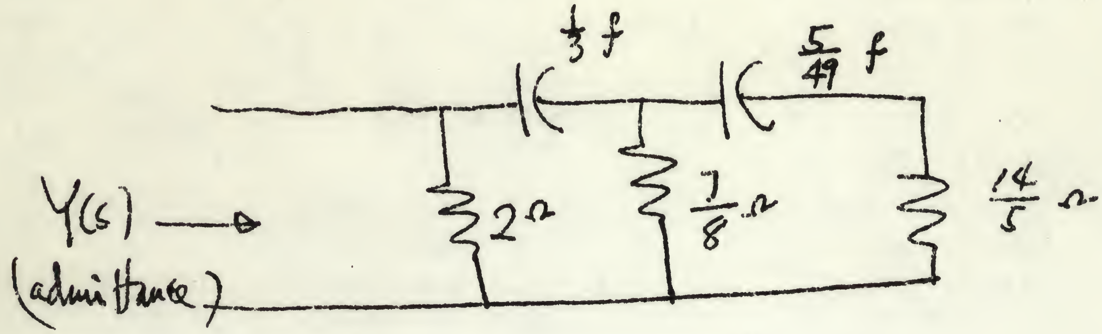
```
           .2000E+01
.1000E+01  .2000E+01


           .6000E+01  .2400E+02
.1000E+01  .1800E+02  .2400E+02


           .1200E+02  .2400E+03  .7200E+03
.1000E+01  .7200E+02  .6000E+03  .7200E+03


           .2000E+02  .1200E+04  .1512E+05  .4032E+05
.1000E+01  .2000E+03  .5400E+04  .3528E+05  .4032E+05


           .3000E+02  .4200E+04  .1411E+06  .1452E+07  .3629E+07
.1000E+01  .4500E+03  .2940E+05  .5645E+06  .3266E+07  .3629E+07


           .6000E+01  .0000E+00
.1000E+01  .6000E+01  .0000E+00


           .4000E+01  .0000E+00
.1000E+01  .4000E+01  .0000E+00


           .1500E+01  .4000E+01
.1000E+01  .8000E+01  .1200E+02


           .0000E+00  .0000E+00
.1000E+01  .0000E+00  .0000E+00
```

$$Y(s) = \frac{1}{2} + s\left(\frac{1.5s+4}{s^2+8s+12}\right) = \frac{6+8s+2s^2}{12+8s+s^2}$$

"exact"

(see next page)

Find the X'fer function of the network:




$$\therefore\ Y(s) = \frac{1}{2} + \cfrac{1}{\frac{3}{s} + \cfrac{1}{\frac{8}{7} + \cfrac{1}{\frac{49}{5s} + \cfrac{1}{5/14}}}}$$

OR

$$= \frac{1}{2} + s\left[\cfrac{1}{3 + \cfrac{1}{\frac{8}{7s} + \cfrac{1}{\frac{49}{5} + \cfrac{1}{\frac{5}{14s}}}}}\right]$$

$$= \frac{1}{2} + s\left(\frac{b_1 s^{n-1} + \cdots + b_n}{s^n + a_1 s^{n-1} + \cdots + a_n}\right)$$

CALL "CFI" Using the following data:

$n = 2$

$h_1 = 3$

$h_2 = \frac{8}{7} \cong 1.1429$

$h_3 = \frac{49}{5} = 9.8$

$h_4 = \frac{5}{14} = 0.3571$

Result

$$= \frac{1}{2} + s\left( \cdots \right) = \frac{6 + 8s + 2s^2}{12 + 8s + s^2}$$