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DECUS NO.	8-567
TITLE	EXPO
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SOURCE LANGUAGE	8K FORTRAN

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SUB 8K-# Jan. 9, 1972

(1) NAME: EXPO

Purpose: To evaluate the approximate state transition matrix/ its augmented transition matrix,

$$e^{AT}$$

where A is a square matrix, and T is a sampling period, which appear usually in the state variable approach to engineering problems,

$$\dot{X}(nT) = A\ X(0)$$

where X(t) is an m x 1 state variable vector at state t, X(0) is the initial values of X(t), and A is in general time dependent state transition matrix of dimension m x m, if the input forcing function is not identically zero, then A is treated as an augmented transition matrix which may or may not be singular, as long as the approach of solution avoid using inverse of A which is what is done in EXPO.

(2) CALLING SEQUENCE:

CALL EXPO(M,T,NPTS,DIGIT)

where;

Input Data:

A(I,J),I,J= 1,2, ...,M (M at most 7)
M = # of row/ # of column/ # of state variables used
T = sampling period
NPTS = # of calculated samples
DIGIT = desired # of significant figures accuracy

Return Data:

AX(I,J), I,J=1,2, ..., M the approximate transition matrix

(3) ERROR RETURN:

"FPNT" ERROR AT LOC 03550 will be printed if computer overflow/underflow.

(4) SPECIAL CONSIDERATION:

If machine overflow/underflow, user is urged to resubmit the program for process with a reduced value of T.

(5) SUBPROGRAM CALLED:

-none-

(6) STORAGE REQUIRED:

pages(octal)

(7) ALGORITHM & REFERENCES:

Algorithm:

$$e^{AT} = M + R$$

where: $M = \sum_{i=0}^{K} \frac{A^i T^i}{i!}$, $A^0 = I$, idnentity matrix

$$R = \sum_{i=K+1}^{\infty} \frac{A^i T^i}{i!} \quad \text{remainder matrix}$$

step1 : Choose K arbitrarily(ref. 3) and evaluate the elements m_{ij} of M.

step2 : Determined e by

$$e = \frac{\|A\| \cdot T}{K+2}$$

where: the linear norm is defined as

$$\|A\| = \max_{i} \left(\sum_{j=1}^{m} |a_{ij}| \right)$$

step3 : Find the upper bound of each element in the remainder matrix,

$$|r_{ij}| \leq \frac{(\|A\| \cdot T)^{K+1}}{(K+1)!} \cdot \frac{1}{1-e}$$

step4 : Compare each m_{ij} from step 1 with the uppoer bound of $|r_{ij}|$ from step 3, if

$\left|r_{ij}\right| \leq 10^{-d} \left|m_{ij}\right|$ where d = accuracy at least "d" significatnt figures is not satisfied, then increase K and repeat step 1 thru step 4, otherwise the process is finished.

References:

1. M.L. Liou, "A Novel Method of Evaluating Transient Responses", Proc. of IEEE, vol. 54, No. 1 Jan. 1966, pp. 20-23

2. J.B. Plant, "On the Computation of Transition Matrices"for Time-Invariant Systems", Proc.IEEE Aug. 1968, pp 1397-1398

3. W. Everling, "On the Evaluation of e^{AT} by power series", Proc. IEEE(letters) 1967, pp. 413

4. FF Kuo, zjJ.F.Kaiser, "System Analysis by Digital Computer", New York, Wiley, 1966, chap.3.

5. S.Ganapathy, A.S. Rao,"Transient Response Evaluation from the State Transition Matrix", Proc. IEEE(letters), march 1969, pp. 347-348.

6. T.A. Bickart,"Matrix Exponential: Approximation by Truncated Power Seires", Proc. IEEE(letters), May 1968, pp. 872-873

7. A.K. Choudhury, D.R. Choudhurry,B.Roy,A.K.Mandal, "On the Evaluation of AT", Proc. IEEE(letters), June 1968, pp. 1110-1111

(8) <u>LISTING:</u>

(see attach)

(9) <u>SAMPLE:</u>

(see attach)

```
C ****  CALCULATION OF APPROXIMATE " STATE TRANSITION MATRIX "
C ****  JAN. 9, 1972
        SUBROUTINE EXPO(M,T,NPTS,DIGIT)
        COMMON A, AX
        DIMENSION A(7,7),AX(7,7),C(7,7),E(7,7)
        KK=4
22      DO 10 I=1,M
        DO 10 J=1,M
        C(I,J)=0.
        AX(I,J)=0.
10      CONTINUE
        DO 20 I=1,M
        AX(I,I)=1.
20      C(I,I)=1.
        AK=1.
        DO 50 II=1,KK
        DO 30 I=1,M
        DO 30 J=1,M
        S=0.
        DO 25 K=1,M
25      S=S+C(I,K)*A(K,J)
        E(I,J)=S
30      CONTINUE
C ..... K  FACTORIALS
        AK=FLOAT(II)*AK
        DO 50 I=1,M
        DO 50 J=1,M
        C(I,J)=T*E(I,J)/AK
        AX(I,J)=AX(I,J)+C(I,J)
50      CONTINUE
        AMAX=0.2E-38
        DO 70 I=1,M
        S=0.
        DO 65 J=1,M
65      S=S+ABS(A(I,J))
        IF(AMAX-S) 55,70,70
55      AMAX=S
70      CONTINUE
C ***   RATIO OF A TERM BY ITS PRECEEDING TERM
        EX=FLOAT(KK+1)
        EPS=AMAX*T/(EX+1.)
C ***   UPPER BOUND OF EACH ELEMENTS IN REMAINDER MATRIX
        R=(EPS*(EX+1.))**EX/((AK*EX)*(1.-EPS))
        DO 80 I=1,M
        DO 80 J=1,M
C ***   ACCURACY UP TO " DIGIT " SIGNIFICANT FIGURES
        IF(R-ABS(AX(I,J))*10.**(-DIGIT)) 80,80,85
85      KK=3*KK/2
        GO TO 22
80      CONTINUE
        RETURN
        END
```

EXPO(M, T, NPTS, DIGIT) Can evaluate numerically the "Inverse Laplace transform", $x(t)$, (i.e. transient response) of

$$X(s) = \frac{a_m s^{m-1} + a_{m-1} s^{m-2} + \cdots + a_2 s + a_1}{s^m + b_m s^{m-1} + b_{m-1} s^{m-2} + \cdots + b_2 s + b_1}$$

with the initial values:

$$x(0+) = a_m$$

$$\dot{x}(0+) = a_{m-1} - b_m x(0+)$$

$$\ddot{x}(0+) = a_{m-2} - b_m \dot{x}(0+) - b_{m-1} x(0+)$$

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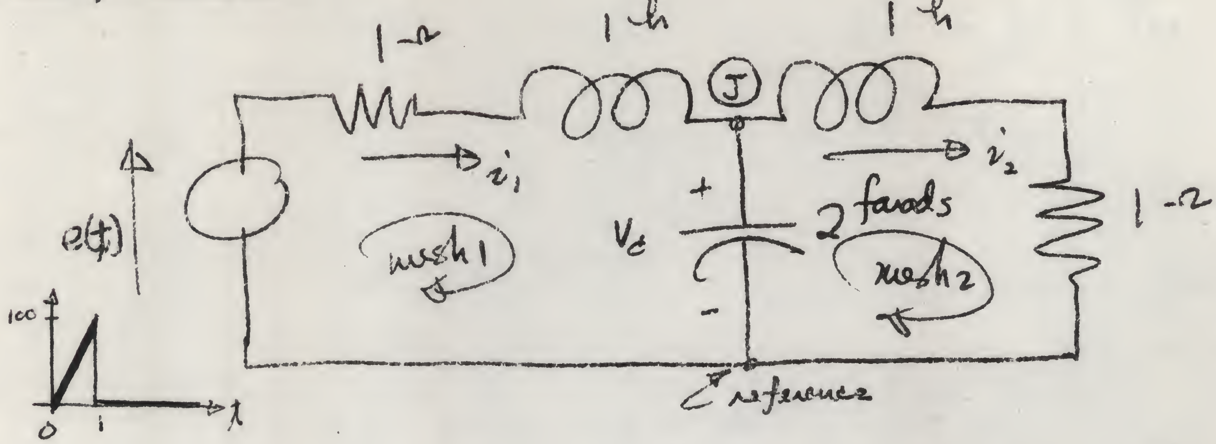
```
C ****   SAMPLE # 3'
C ****   INVERSE LAPLACE THANSFORM OF A HATIONAL FUNCTION
C
C ****   A IS AN  M BY M STATE TRANSITION MATRIX
C        M   = # OF STATE VARIABLES
C        NT  = TOTAL # OF PTS. TO BE PUNCH OUT
C        TMX = MAX. TIME
C        TSC = TIME SAMPLING PERIOD CALC  ( IN "EXPO"  ) MUST
C              BE LESS THAN OR EQUAL TO (TMX/(NT-1))
C
         COMMON A, C
         DIMENSION A(7,7),C(7,7),X(7)
         READ(2,5) M, NT,TMX,TSC,DIGIT
5        FORMAT(I10/I10/E12.6/E12.6/E12.6)
         NP=TMX/FLOAT(NT-1)/TSC
         NC=NP*NT
         DO 30 I=1,M
         DO 20 J=1,M
20       READ(2,10) A(I,J),
         FINI
30       CONTINUE
10       FORMAT(E10.4)
C ***    READ STATE VARIABLES INITIAL VALUES
         DO 25 I=1,M
25       READ(2,10) X(I)
C
         CALL EXPO(M,TSC,NC,DIGIT)
         T2=0.
         DO 320 N=1,NC
C ***    PRINT TIME, TRANSIENT SOLUTION, FIRST DERIVATIVE
         IF(IREM((N-1)/NP)) 316,312,316
312      WRITE(2,315) T2,X(1),X(2)
315      FORMAT(3E13.4)
316      DO 310 I=1,M
         S=0.
         DO 300 J=1,M
300      S=S+C(I,J)*X(J)
         X(I)=S
310      CONTINUE
         T2=T2+TSC
320      CONTINUE
         STOP
         END
```

```
 C ****  A IS AN  M BY M STATE TRANSITION MATRIX
C
        COMMON A, C
        DIMENSION A(7,7),C(7,7),X(7)
        READ(2,5) M, NPTS, T, DIGIT
5       FORMAT(I1Ø/I1Ø/E12.6/E12.6)
        DO 3Ø I=1,M
        DO 2Ø J=1,M
2Ø      READ(2,1Ø) A(I,J),
        FINI
3Ø      CONTINUE
1Ø      FORMAT(E1Ø.4)
C ***   READ STATE VARIABLES INITIAL VALUES
        DO 25 I=1,M
25      READ(2,1Ø) X(I)
C
        CALL EXPO(M,T,NPTS,DIGIT)
        T2=Ø.
        DO 32Ø N=1,NPTS
C ***   PRINT TIME, TRANSIENT SOLUTION, FIRST DERIVATIVE
        WRITE(2,315) T2,X(1),X(2)
315     FORMAT(3E13.4)
        DO 31Ø I=1,M
        S=Ø.
        DO 3ØØ J=1,M
3ØØ     S=S+C(I,J)*X(J)
        X(I)=S
31Ø     CONTINUE
        T2=T2+T
32Ø     CONTINUE
        STOP
        END
```

Sample #2′ : 3rd - Order Butterworth Filter



Obtain & Plot i_1, i_2 vs t over $0 \le t \le 10$ sec. with 50 pts., with sampling period = .01 sec

Solution: By mesh & node methods we have:

mesh 1 $\quad i_1' = -i_1 - v_c + e(t)$

mesh 2 $\quad i_2' = -i_2 + v_c$

node (J) $\quad v_c' = 0.5\,i_1 - 0.5\,i_2$

Let "State Variable

$$X = \begin{bmatrix} i_2 \\ i_1 \\ v_c \\ e(t) \end{bmatrix}$$

and

Augmented State transition matrix

$$A = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 1 \\ -0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

then $X' = AX$

whose solution is $X(nT) = e^{AT} X(0)$ with $X(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \textcircled{0} \end{bmatrix}$

```
C ****  SAMPLE # 5  (WILL OUTPUT:  TIME, INPUT, RESPONSE )
C       CALL:  EXPO   &  INPUT  SUBROUTINES
C ****  SOLVING:
C       X' = A*X + B*U
C WHERE X IS A STATE VARIABLES
C       A IS TRANSITION MATRIX
C       B ASSOCIATED MATRIX TO INPUT FORCING FUNCTION  U
C
C       MUST CONVERT TO:
C
C       X' = A*X    WHERE  A  IS NOW AN AUGMENTED MATRIX
C
C
C ****  A IS AN  M BY M  "AUGMENTED TRANSITION MATRIX"
C       M   = # OF STATE VARIABLES
C       NT  = TOTAL # OF PTS. TO BE PUNCH OUT (PER VARIABLE)
C       TMX = MAX. TIME
C       TSC = TIME SAMPLING PERIOD CALC  ( IN "EXPO"  ) MUST
C             BE LESS THAN OR EQUAL TO (TMX/(NT-1))
C
        COMMON A, C
        DIMENSION A(7,7),C(7,7),X(7)
        READ(2,5) M, NT,TMX,TSC,DIGIT
5       FORMAT(I10/I10/E12.6/E12.6/E12.6)
        NP=TMX/FLOAT(NT-1)/TSC
        NC=NP*NT
        DO 30 I=1,M
        DO 20 J=1,M
20      READ(2,10) A(I,J),
        FINI
30      CONTINUE
10      FORMAT(E10.4)
C ****  READ STATE VARIABLES INITIAL VALUES
        DO 25 I=1,M
25      READ(2,10) X(I)
C
        CALL EXPO(M,TSC,NC,DIGIT)
        T2=0.
        DO 320 N=1,NC
C ****  PRINT TIME, INPUT FCN, TIME RESPONSE
        IF(IREM((N-1)/NP)) 316,312,316
312     WRITE(2,315) T2,X(M),X(1)
315     FORMAT(3E13.4)
316     DO 310 I=1,M
        S=0.
        DO 300 J=1,M
300     S=S+C(I,J)*X(J)
        X(I)=S
310     CONTINUE
        T2=T2+TSC
C
        CALL INPUT(X4,T2)
C ****  SET INPUT AS LAST VARIABLE
        X(M)=X4
320     CONTINUE
        STOP
        END
```

```
C ****  INPUT # 2
C       E(T) = 100*T,   FOR 0<= T <=1.
C            = 0        FOR  T> 1.
C
        SUBROUTINE INPUT(X4,T2)
        IF(T2-1.) 5,5,10
5       X4=100.*T2
        RETURN
10      X4=0.
        RETURN
        END
```