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DECUS NO.	FOCAL8-120
TITLE	PFI - PRODUCT FORM OF THE INVERSE
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PFI - PRODUCT FORM OF THE INVERSE

DECUS Program Library Write-up

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INTRODUCTION

The product form of the Inverse (PFI) provides a means of inverting a matrix with minimal storage and computation requirements. It is based on the fact that, if we already have the inverse B of an (nxn) matrix A, we can obtain the inverse $\hat{B}$ of a new matrix $\hat{A}$, formed by replacing column r of A by a vector $\underline{x}$, with the formulae:

$$\left.\begin{aligned} \hat{b}_{ij} &= b_{ij} - \frac{y_i}{y_r} b_{rj}, \quad i \neq r \\ \hat{b}_{rj} &= b_{rj} / y_r \end{aligned}\right\} \qquad j = 1, \ldots, n,$$

where $\underline{y} = B\underline{x}$. Thus, to obtain A^{-1}, we begin by setting B equal to the identity matrix (or any non-singular matrix), and perform the PFI n times, once for each column of A. We note that this procedure requires storage only for the current value of B and the vector $\underline{y}$; the components of $\underline{x}$ are used only to compute a new $\underline{y}$ (in lines 2.2, 6.3 and 6.4). Note also that the subscripting algorithm B(I, J) = B(I+N * (J-1)) requires few actual multiplications to evaluate the current subscript.

A typical encounter is shown, in which the matrix

$$B = \begin{bmatrix} 1 & 0 & -1 \\ .2 & -1 & 0 \\ 0 & .75 & -1 \end{bmatrix}^{-1}$$

is evaluated; note that a negative column number input gives a printout of the current B matrix. The second example obtains the inverse of

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & .05 & -1 \end{bmatrix}$$

by simply altering the first two columns, while the last example inverts

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & .9 & -1 \end{bmatrix}$$

by altering column 2 only. The ability to do this is especially useful when doing parametric studies or linear programming.

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C FOCAL V3A
01.01C  JAMES H. CHRISTENSEN
01.02C  SCHOOL OF CHEM.ENG.& MATLS.SCI.
01.03C  UNIV. OF OKLA.
01.04C  202 W. BOYD #23
01.05C  NORMAN ,OK. 73069
01.10  T  !!!, "PROGRAM TO INVERT A MATRIX USING PRODUCT FORM",!
01.20  A "DIMENSION OF MATRIX" N
01.30  F  I=1,N*N;S B(I)=0
01.35  F  I=1,N;S B(I+N*I-N)=1
01.40  A "COLUMN"R; I (R) 1.5,1.5;D 2.0;G 1.4
01.50  T  !!,"INVERSE IS:"
01.60  F  I=1,N;T !;F J=1,N;T %7.06,"  ",B(N*J-N+I)
01.70  T  !!;G 1.4
02.10  F  J=1,N;S Y(J)=0
02.20  S  ISB=-N;F J=1,N;D 6.3
02.30  I  (FABS(Y(R) )-1.E-10) 6.2;S YR1=1/Y(R)
02.40  S  ISB=-N;F J=1,N;S ISB=ISB+N;S BRJ=YR1*B(ISB+R);D 3.0
03.10  F  I=1,N;S B(ISB+I)=B(ISB+I)-Y(I)*BRJ
03.20  S  B(ISB+R)=BRJ
06.20  T  !,"MATRIX IS SINGULAR",!!;R
06.30  S  ISB=ISB+N;A X;I (X)6.4,6.5,6.4
06.40  F  I=1,N;S Y(I)=Y(I)+X*B(ISB+I)
06.50  RETURN

*;
*
```

```
PROGRAM TO INVERT A MATRIX USING PRODUCT FORM
DIMENSION OF MATRIX:3
COLUMN:1:1:.2:0
COLUMN:2:0:-1:.75
COLUMN:3:-1:0:-1
COLUMN:-1

INVERSE IS:
     1.176471       -0.882353       -1.176471
     0.235294       -1.176471       -0.235294
     0.176471       -0.882353       -1.176471

COLUMN:1:1:1:0
COLUMN:2:0:-1:.05
COLUMN:-1

INVERSE IS:
     1.052632       -0.052632       -1.052632
     1.052632       -1.052632       -1.052632
     0.052632       -0.052632       -1.052632

COLUMN:2:0:-1:.9
COLUMN:-1

INVERSE IS:
     10.00000       -9.000000       -10.00000
     10.00000       -10.00000       -10.00000
     9.000000       -9.000000       -10.00000

COLUMN: ↑ P
FOCAL V4A
*W
```